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THE ESTIMATION OF THE TEMPERATURE PROFILE IN A LAMINAR BOUNDARY LAYER WITH A SCHLIEREN METHOD

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Abstract—Schlieren methods for heat-transfer measurement make use of ray deflections due to gradients of refractive index. The analysis of ray deflections is recapitulated briefly, and an experimental technique is described. A comparison between measured ray deflections and those predicted from analysis is made for natural convection over a horizontal heated cylinder, and good agreement is found. A transcendental temperature profile approximation function is described, and it is shown that this also gives good quantitative agreement with measured ray deflections. Finally, procedures are given by which it is possible to estimate the distribution of temperature and temperature gradient in a thermal boundary layer from ray deflection measurements, making use of the approximation function. The optical technique is established as a practical tool for quantitative study of temperature profiles in laminar boundary layers.

NOMENCLATURE

- a, b, parameters of the transcendental profile approximation function. Numerical values quoted are associated with use of η as the argument, and therefore are dimensionless;
- B, atmospheric pressure [mm Hg];
- c, local velocity of light;
- c_0, velocity of light *in vacuo*;
- K, curvature;
- l, distance traversed by light rays in a region of finite refractive index gradient;
- L, distance traversed by light rays after passing through region of finite refractive index gradient to observation position;
- m, ray-deflection coefficient, characteristic of a specific optical configuration;
- n, refractive index;
- N, direction normal to a light ray in the direction of its curvature;
- P, region through which rays pass after l-region;
- R, cylinder radius;
- t, time;
- T, absolute temperature: $T = T(y, \zeta)$ or $T = T(x, y)$;
- T_0, absolute temperature of ambient air;
- T_1, $T_W - T_0$;
- T_W, absolute temperature of body surface;
- x, space co-ordinate on body in direction of flow;
- y, ray position above body surface; post-superscript primes denote differentiation with respect to ξ: (later) space co-ordinate normal to body surface;
- δ, ray position: with subscript L, ray position at observation plane;
- Δ, ray deflection: with subscript L, ray deflection at observation plane;
- ζ, space co-ordinate orthogonal to light ray and lying in local isochoric surface;
- η, dimensionless space co-ordinate,

$$\eta = \frac{y}{R} N_{Gr}^{1/4}$$

- ξ, space co-ordinate in direction of light ray as it enters the l-region;
- ρ, density;
- N_{Gr}, Grashof number based on radius. Value is $0{\cdot}73 \times 10^5$ for experiment on cylinder;
- pro, profile function; post-superscript primes denote differentiation.

1. INTRODUCTION

THE GENERAL characteristics of Schmidt's optical method for determination of local heat-transfer coefficients over the surface of a plane or cylindrical body are widely known, having been described in several books e.g. [1]. This is a schlieren method which uses the caustic line to estimate the wall gradient of temperature and does not give information about the temperature field. The manner in which the measurements can be interpreted quantitatively is perhaps less well known, and it does not appear to have been generally appreciated that it is possible—by a slight modification of the experimental arrangement—to estimate the local temperature profile in the thermal boundary layer. In a recent paper on some optical properties of thermal boundary layers, Grigull [2] has given a clear review of the analysis for light ray deflection and demonstrated a comparison of the ray deflection pattern expected from a specific temperature distribution with the actual ray deflection pattern. For illustrative purposes the temperature profile was represented by a polynomial of the fourth degree. The deflection of light rays entering the boundary layer at different levels was made apparent by using a restricted light beam which had initially a cross-section bounded by an isosceles triangle. This technique represents a significant advance in the use of the optical method in boundary layers.

It should be mentioned that optical techniques have been developed, for studies of other phenomena occurring in flows, which are closely related to the technique described here. In this connection, Weinberg's book on the optical properties of flames [3] merits attention. As a historical matter, it appears that Dvorak [4] was the first to report results of the projection of near-parallel light through a temperature field.

In the process of some research upon the influence of sound fields on convection from cylinders, it became necessary to examine the inverse problem: to estimate temperature distribution in thermal boundary layers by observing the deflection of rays at various positions within the boundary layer. It was desired to do this with the minimum of interference with the total light field since it was also necessary to measure the local heat transfer at all positions around the cylinder. It proved necessary to make such observations not only to determine local temperature profiles but also to avoid ambiguities in the interpretation of the local heat-transfer coefficient from the ordinary deflected light patterns.

In this paper, the relationship of the deflection of light rays to conditions in the thermal boundary layer is recapitulated briefly. A simple experimental technique, related closely to that described by Grigull, is then described by which the deflection of rays at various positions within the boundary layer can be measured. Since this deflected ray pattern is determined experimentally, the uncertainties attendant upon all physical measurements preclude the possibility of recovering from it the exact boundary layer temperature distribution. However, it seems reasonable to approximate the exact distribution by a member of a suitable family of approximating functions, and statistically fit the deflected ray pattern into the family. In this sense it is possible to estimate the temperature distribution, and a suitable family of approximating functions, involving only two adjustable constants, is described below, together with its application to the current problem.

2. THE DEFLECTION OF LIGHT RAYS

When a ray of light passes into a region of variable density, and consequently of variable refractive index n, it is deflected from a straight path. If the region is sufficiently deep it is possible for some rays to undergo reflection, in the sense that $\mathbf{c}\cdot \text{grad}\, n$ can change sign from negative to positive as one follows the ray trajectory.

A basic relationship which must be satisfied is

$$cn = c_0 \qquad (1)$$

where c and c_0 are respectively the magnitudes of the local velocity of light and the velocity of light *in vacuo*. It is apparent that the velocity of light will vary along a ray if the latter passes through a region of variable refractive index.

For the present application, it is not necessary to be concerned with the velocity distribution of light, since the relative phases of different rays are not significant to the technique involved here. The analysis of the ray deflection was given in detail by Schmidt [5] in 1932 and reviewed by

Grigull, but it is worthwhile to mention the essential features and assumptions here.

A ray of light is considered as it enters a region of variable refractive index n, with $n = n(\rho)$. A co-ordinate system can be established with the ξ-direction tangential to the ray as it enters the region and the ζ-direction tangential to the local isochoric surface. If there is negligible torsion of the ray over an interval near the origin, then within this region the trajectory of the ray is $y = y(\xi)$ and the local curvature is

$$K = y''/(1 + y'^2)^{3/2} \tag{2}$$

where the positive square root is taken. The ray can be thought of as normal to the wavefronts of light as they progress through the region. Since the velocity of light is not uniform across the wavefront in the region considered, a rotation of the wavefront occurs which is directly related to the transverse gradient of refractive index. Thus, in a time interval $\mathrm{d}t$ a point on a wavefront advances $c\ \mathrm{d}t$ while a neighboring point which is $\mathrm{d}N$ closer to the center of curvature advances

$$\left\{c + \left(\frac{\mathrm{d}c}{\mathrm{d}N}\right)\mathrm{d}N\right\}\mathrm{d}t.$$

This corresponds to an angular rotation of $-(\mathrm{d}c/\mathrm{d}N)\,\mathrm{d}t$ in traversing the distance $c\ \mathrm{d}t$, so that from the definition of curvature

$$K = -\frac{1}{c}\frac{\mathrm{d}c}{\mathrm{d}N}.$$

This is related to the transverse gradient of refractive index, from equation (1), by

$$K = \frac{1}{n}\frac{\mathrm{d}N}{\mathrm{d}n} = \frac{\mathrm{d}(\ln n)}{\mathrm{d}T}\frac{\mathrm{d}T}{\mathrm{d}N} \tag{3}$$

It is found that in air, with the temperature gradients typical of convection, K is very small. On recalling the manner in which the co-ordinate system is established, it is possible to write

$$y'' = \frac{1}{n}\frac{\mathrm{d}n}{\mathrm{d}T}\frac{\partial T}{\partial y}$$

and for small values of y, the value of y' after the ray has traversed a length l is

$$y' = \frac{1}{n}\frac{\mathrm{d}n}{\mathrm{d}T}\frac{\partial T}{\partial y}l \tag{4}$$

and the deflection y at l is

$$y = \frac{1}{n}\frac{\mathrm{d}n}{\mathrm{d}T}\frac{\partial T}{\partial y}\frac{l^2}{2}. \tag{5}$$

If the ray passes, at l, into a region P of constant refractive index n_P, then by Fermat's principle

$$y'_P = (n_l/n_P)y'_l. \tag{6}$$

If the ray traverses a distance L in region P, the deflection of the ray from its original position is

$$\Delta_L = \frac{1}{n}\frac{\mathrm{d}n}{\mathrm{d}T}\frac{\partial T}{\partial y}\left\{l\left(\frac{l}{2} + \frac{n_l}{n_P}L\right)\right\}. \tag{7}$$

For air, at temperature T°K and pressure B mm Hg,

$$n = 1 + \frac{0{\cdot}0800}{T}\cdot\frac{B}{760} \tag{8}$$

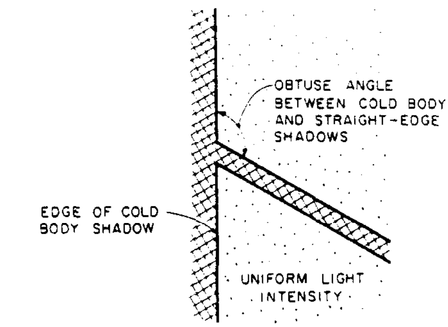
Thus, for air at 760 mm Hg and 300°K, $n = 1{\cdot}000267$, and at 373°K, $n = 1{\cdot}000214$, so that for this temperature range $(n_l/n_P) = 0{\cdot}999947$. It is clear that for most temperatures of interest in air, (n_l/n_P) can be ignored as a factor. This consideration can be applied retroactively, so to speak, to the rays in passing from the region optically upstream of the l-region into the l-region. In use of the optical method, it is usually possible to arrange for the light rays to pass (initially) parallel to a heated solid surface over which the temperature distribution is constant in the direction of the rays. This involves use of a configuration where the flow is two-dimensional, e.g. laminar flow on a plate or a cylinder transverse to the flow, with the rays directed across the flow.

From this point on, it will be convenient to ascribe the symbol y as a space co-ordinate with its origin at the body surface instead of imbedded in any ray as the latter enters the l-region. For a ray which enters the region of l at a level y above the heated body surface, the deflection is

$$\Delta_L(y) = -\frac{0{\cdot}0800}{\{T(y)\}^2}\frac{Bl}{760}\{L + l/2\}\frac{\mathrm{d}T}{\mathrm{d}y}(y) \tag{9}$$

and the total distance δ_L of the ray at L from the baseline is $y + \Delta_L(y)$.

The distance L is essentially the station at which the ray position pattern is observed. It is worthwhile to note that a distinction is made between the ray deflection pattern and the ray position pattern. For values of L less than a critical value, the set of rays remains ordered in y, i.e. the rays do not intersect. The critical value of L occurs when $d\Delta_L/dy = -1$ at some value of y. The distance at which this occurs is clearly very sensitive to the local boundary layer thickness and to the temperature profile—especially its second derivative. The critical distance can range typically from a few inches to several feet.



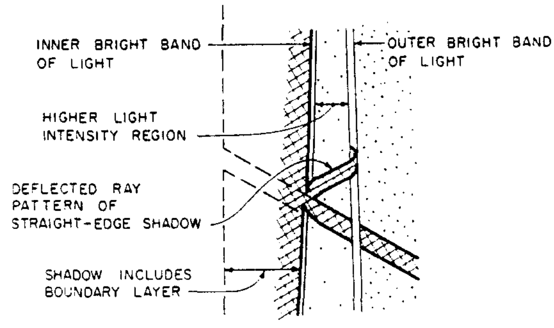


FIG. 1. The position of particular rays which have passed through a thermal boundary layer can be determined from the deflected ray pattern of a straight-edged body placed optically upstream of the heated body.

3. A SIMPLE EXPERIMENTAL TECHNIQUE

To obtain a measurement of the deflection of light rays at different distances from the heated surface, it is necessary to make some arrangement whereby rays can be labelled, so to speak, and still be visible after deflection. A very simple arrangement by which this can be accomplished is to insert into the light beam, just before it passes over the heated body, a body for which the shadow of the edge lies in a clearly defined position across the boundary layer. For example, a straight-edged body can be arranged such that the shadow of its straight edge makes an obtuse angle with the shadow of the flat surface of the convection body when the latter is unheated. Upon heating of the convection body, the rays which define the shadow of the straight edge within the boundary layer are re-mapped, as sketched in Fig. 1. The similarity of this ray position pattern to that illustrated in Fig. 9 of Grigull's paper is obvious. In practice, one of the problems which must clearly be faced is the choice of the angle of the straight-edge to the convection surface. If the angle of deflection of the edge from the geometric normal is small, then there is considerable uncertainty in determining the deflection of rays at specific distances, in the boundary layer, from the solid surface, since all lines are to *some* degree fuzzy when all photographic recording and developing has been completed. If the angle of deflection of the edge from the geometric normal is large, then the profile which is being estimated is some sort of mean over a length of the surface in the direction of fluid flow, rather than the profile at a particular value of the longitudinal co-ordinate measured on the convection surface in the direction of the external flow. The choice is not a difficult one to make: since it is characteristic of boundary layers that longitudinal gradients are far smaller than transverse gradients (provided that the longitudinal gradient of surface temperature is not large) no serious errors are generally introduced when fairly large deflections are used. On this basis angles of 30° or so to the normal can easily be justified.

There remains the practical question of what form of straight-edged body is most useful for this purpose while obstructing as little of the light as possible. It appears that satisfactory results can be obtained with the use of stretched wires having diameters of the order of 0·020 in, in conjunction with light from a point source distant about 40 ft from the convection body and with the viewing screen (or photographic plate) about 8 ft from the body.

4. APPROXIMATING FUNCTION FOR TEMPERATURE DISTRIBUTION

Over the last 40 years or so, much experience has been gained in the use of low-order polynomials as approximations for velocity and temperature profiles in boundary layers. If one accepts that a fourth degree polynomial will give sufficient accuracy, then there are five constants (the coefficients in the polynomial) to be determined. On the other hand, a family of transcendental approximations has more recently beentested success fully for boundary-layer profiles [6], and this family has only two constants to be determined. Further, the transcendental profile and all its derivatives are continuous.

The temperature profile to be approximated exists between absolute temperatures T_W at the convection body surface and T_0 outside the boundary layer. It is possible to write

$$T(x, y) = T_0 + (T_W(x) - T_0) \text{ pro } (x, y) = T_0 + T_1(x) \text{ pro } (x, y) \qquad (10)$$

where pro (x, y) is the appropriate profile function. The transcendental approximation profile, mentioned above is

$$\text{pro } (x, y) = \frac{\exp \{- \exp [a(x) + b(x)y]\}}{\exp [- \exp a(x)]} \qquad (11)$$

for which

$$\frac{\partial}{\partial y} (\text{pro } y) = b \text{ pro } y \{\ln (\text{pro } y) - \exp a\}.$$

It can be shown that

$$\Delta_L = \delta_L - y = - \frac{m b T_1 \text{ pro } y \exp (a + by)}{\{T_0 + T_1 \text{ pro } y\}^2} \qquad (12)$$

where

$$m = - \frac{0{\cdot}0800\, Bl(L + l/2)}{760}.$$

When large supercritical distances L are used in experiments, the observed ray position pattern contains a band of rays which have large deflections [see, for example, Figs. 9(f), (g) and (h) of Grigull's paper]. These bands are genuine effects of the optical ray position pattern, and their finite width does not mean that poor photographic work has been done. In order to measure the heat-transfer coefficient, it is necessary to determine the position of the ray, within the band, which originally passed immediately adjacent to the surface. This position need not be at the outer edge or at the middle of the band, and uncertainty about the ray position introduces an uncertainty of the order of 7 per cent into heat-transfer measurements made in this way. $d\delta_L/dy$ is negative over a major portion of the boundary layer, but it is possible for it to be positive at very small y. When this is so, the ray corresponding to $y = 0$ at the heated surface lies within the bright band, and its position has to be identified with aid of knowledge of the temperature distribution.

5. COMPARISON WITH A KNOWN TEMPERATURE PROFILE

In order to assess the accuracy of the optical method and the degree to which a typical real temperature profile can be represented by the profile function, an experiment was performed. The heat-transferring body was a horizontal brass cylinder, in which a two-phase water-vapor-water system could be maintained at atmospheric pressure, and over which natural convection of air was permitted. A water-cooled 1 kW mercury arc lamp with a 0·017 in diameter aperture was used as the light source. Rays from this source underwent reflection from two plane first-surface mirrors and passed through one plane glass window before passing over the cylinder, through a second plane glass window and onto a film. The optical path from source to cylinder was approximately 40 ft. The cylinder was tapered slightly to match the light rays; the mean diameter of the cylinder was 1·830 in. The cylinder was located in an anechoic chamber (since this present investigation arose in connection with measurements of the influence of sound on heat transfer), and the glass windows were in the walls of this chamber. The chamber interior was roughly an 8 ft cube, so that no closed-space effects need be expected on the 18 in length cylinder. Photographs were taken on Kodak Plus-X film pack, 5 by 7 in sheets, with exposures of about 7 s; fine-grain development was used. Approximately $1\frac{1}{2}$ in optically upstream of the thin end of the cylinder a brass disk-and-ring former was supported with its axis coincident with that of the cylinder. Between the disk and ring several wires were stretched so that each was skew from the radial direction by the order of 0·5 radians.

A photographic negative was obtained for natural convection (in the absence of sound fields) from the cylinder, and a wire shadow located at 0·5 radians round from the bottom of the cylinder was arbitrarily selected for analysis. Measurements of Δ_L were made from this negative, with the aid of a polar plotter, for rays which which entered the thermal boundary at various levels y. Chiang and Kaye [7] have presented an analysis for natural convection over a horizontal cylinder; their paper includes tables for the temperature profile and its first derivative (to which ray deflection is directly related), and makes it possible to tabulate directly these quantities at any circumferential position around the cylinder. The transformed distance normal to the cylinder, denoted here by η, is identical with the variable y used by Chiang and Kaye. Measurements of the ray deflection pattern Δ_L and values predicted from the Chiang and Kaye tables are plotted in Fig. 2. It can be seen that agreement is very satisfactory; the differences lie within the uncertainties associated with the measurements and with the analysis. It may be noted that the Chiang and Kaye analysis is based upon the boundary-layer equations set in orthogonal curvilinear co-ordinates; as such, the analysis is asymptotically valid at large Grashof numbers. As the Grashof number decreases, the laminar boundary layer becomes physically larger compared with the cylinder—in the present case, the boundary layer is about 0·3 in thick—and the conditions assumed for the analysis are not satisfied. It is not known quantitatively how significant this is to the temperature profile. The deflections Δ_L can be measured with an uncertainty of about 0·02 in, and the location of the zero of the η scale is uncertain by about 0·1 (dimensionless).

The ray position profile corresponding to the Chiang and Kaye analysis is plotted in Fig. 3. From this it can be seen that the ray position

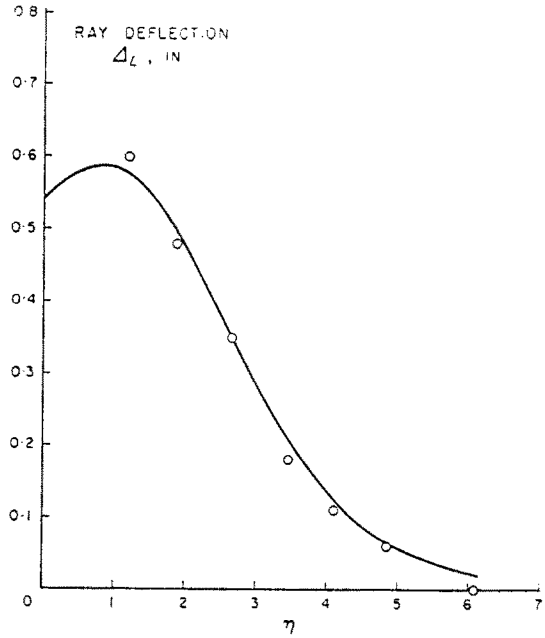


FIG. 2. Ray deflection pattern for natural convection on a heated horizontal cylinder. The solid line shows the predicted deflection pattern derived from the solution of Chiang and Kaye; the points are observed deflections.

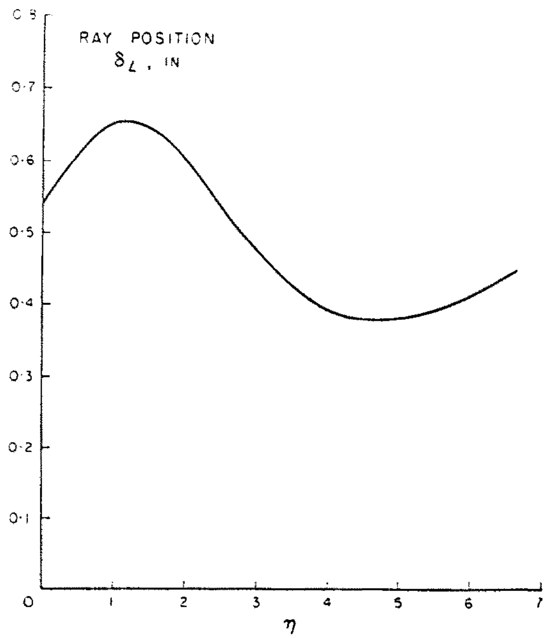


FIG. 3. Ray position pattern derived from the solution of Chiang and Kaye.

corresponding to $y = 0$ at the heated surface is not the extreme ray in the bright-band region.

Values of a and b for the profile function, equation (11), were computed on a trial-and-error basis to have exactly the same wall gradient of temperature as Chiang and Kaye, with b determined to the nearest 0·01 to give a close fit to the $\Delta_L(\eta)$ profile derived from Chiang and Kaye's solution. The values found were $b = 0{\cdot}36$, $\exp(a) = 1{\cdot}017$. A graph of $\Delta_L(\eta)$ from this profile function is shown in Fig. 4, together with experimental points.

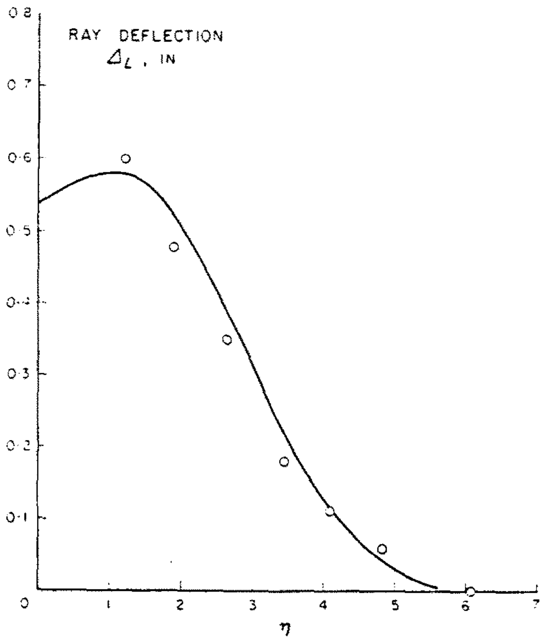


FIG. 4. Ray deflection pattern: the solid line shows the deflection pattern predicted with the transcendental approximation function fitted to the solution of Chiang and Kaye, and the points are observed deflections.

Again, agreement is good, the differences lying with the uncertainties associated with experiment and analysis. It should be pointed out that a comparison on the basis of Δ_L is very cruel to an approximate profile, since it demands good representation not only of the profile but also of its first derivative, and this to quite small values of the temperature: thus, at $\eta = 4{\cdot}0$, the temperature difference has fallen to about 7 per cent of its maximum, yet the Δ_L from the approximation differs from that derived from Chiang and Kaye by only 2 per cent. For this case, the differences in the Δ_L do not exceed 1/30 in, and the differences in the dimensionless temperature do not exceed about 0·03. This degree of agreement is far better than would be attained using a low-order polynomial, and is a consequence of the use of a transcendental profile approximation function. A test of this type is necessary because a direct comparison of $T(y)$ from analysis and from the approximation function makes them look extraordinarily close.

It can be concluded that the ray deflection pattern due to a laminar thermal boundary layer as predicted by theory is in good quantitative agreement with experiment, and that the corresponding temperature profile can be represented well by a two-parameter transcendental approximation.

6. ESTIMATION OF TEMPERATURE PROFILE FROM RAY DEFLECTION MEASUREMENTS

In the previous section, it was demonstrated that a ray deflection pattern predicted from analysis was in good quantitative agreement with experiment. This clearly opens the possibility of estimating temperature profiles from ray deflection measurements. It is important to know what techniques of interpretation are particularly useful, what handicaps are encountered in practice, and what accuracy can be expected.

It proves fairly easy to obtain measurements of Δ_L from photographic negatives, subject to an uncertainty of about 0·02 in due to the thickness of the lines involved. It should, perhaps, be pointed out that the wires used to produce shadows were only slightly larger than the aperture of the source and did not produce single shadows but diffraction bands, the central dark band being used for the measurements. It is also fairly easy to determine the difference in y between each Δ_L measurement. However, it is far more difficult to determine from the negative the true zero for y with an accuracy comparable to distances between Δ_L measurements. This difficulty arises because it requires identification of the position on the negative corresponding to the axis of the cylinder with an uncertainty of less than 0·007 in (this is roughly equivalent to an uncertainty of 0·10 in η for the example discussed in the previous section). It is possible to use certain techniques to overlay two negatives rather precisely, but these are not always reliable and always tiresome. Fortunately, it is possible to avoid these by use of a technique of interpretation which also provides a test of consistency of the ray deflection measurements. It may be recalled that the ray deflection is given by

$$\Delta_L = \frac{mT_1 \text{ pro}'(y)}{\{T_0 + T_1 \text{ pro}(y)\}^2} \qquad (13)$$

where pro (y) is understood as a profile function, not a specified approximation to it. Clearly,

$$\text{pro}'(y) = \frac{1}{mT_1}\{T_0 + T_1 \text{ pro}(y)\}^2 \Delta_L \qquad (14)$$

so that

$$\text{pro}(y) = \frac{1}{mT_1}\int_{\infty}^{y} \{T_0 + T_1 \text{ pro}(y)\}^2 \Delta_L \, dy. \qquad (15)$$

Beginning at large y, this integral can be evaluated numerically from the experimental Δ_L measurements, using a nominal y which differs from the true y by a constant. When pro (y) becomes equal to unity then, from the definition of pro (y), *that* y is the true zero for the y scale. The test of consistency lies in the observation that if one passes pro $(y) = 1{\cdot}0$ and still has Δ_L measurements for yet lower values of y, or if one exhausts Δ_L measurements and yet is far from pro $(y) = 1{\cdot}0$, then some error has been made. Under normal circumstances, it is not possible to make Δ_L measurements right to the end of the wire shadow in the bright band, because it is somewhat indistinct there, so that slight extrapolation is required. However, it appears generally possible to reach to values of y for which pro $'(y)$ has come with 5 per cent or so of its wall value, and extrapolation does not incur a large uncertainty in the position of the true zero of y. The zero for the $\Delta_L(\eta)$ measurements plotted in the previous section was determined by this technique, and fell within 0·02 in of the position estimated for the cold cylinder image, calculated from ray optics. This technique of numerical integration of the ray deflection pattern can be completed quite rapidly, and provides in itself a convenient first approximation to the temperature profile. For estimation of heat transfer it is not adequate, because the process of extrapolation at small y introduces a large uncertainty in the slope at $y = 0$, and no improvement is obtained over the uncertainty from using, say, the inner or outer edge of the bright band in the ray position pattern as the ray position from $y = 0$.

At this juncture it is necessary to use information available in the $\Delta_L(y)$ distribution other than its integral, if an improvement in the estimation of the temperature profile is to be found. This other information is its shape. From the bright band, or from the ray deflection integration, it is possible to establish bounds upon pro $'(0)$. As a practical procedure for evaluation, it is convenient to divide the interval of pro $'(0)$, defined by the bounds, into a finite set of trial values of pro $'(0)$, each differing from its neighbor by 2 or 3 per cent (or whatever difference is felt to be within the resolution capacity of the particular experiment). For each trial value of pro $'(0)$ values for the constants a and b can be determined for the approximation function, equation (11), which give the best fit of the corresponding predicted ray deflection pattern with the experimental values. The best fit for each of the trial values of pro $'(0)$ can then be compared with the others, and the best best-fit selected. In selecting good fits it is important to observe carefully the *distribution* of deviations between each approximation and the experimental points: numerical discriminants which ignore the distribution of deviations, such as the total variance (i.e. square of the standard deviation), are not sufficiently sensitive. For a given trial value of pro $'(0)$, (b exp a) is fixed, and the search for a best fit can be conducted by variation of b. A rapid initial search with b can be made by seeking a value of b which fits Δ_L well in the region of the values of y at which pro (y) is 0·05 and 0·5. At these values the variation with b of the predicted values of Δ_L is nearly linear, and a rapidly convergent iteration of b can be made.

An examination of the deflection pattern described in the previous section, considered on this basis, gave an estimation of the dimensionless temperature gradient at the wall of $0{\cdot}375 \pm 0{\cdot}015$. This compares well with the value 0·3662 given by Chiang and Kaye's analysis.

It is not possible to give a general estimate of the accuracy which can be obtained by use of the methods described here, since the accuracy is sensitive to the circumstances involved. Under favorable circumstances, it appears possible to achieve uncertainties in heat-transfer coefficient of about 3 per cent, and to find a corresponding approximate representation of the temperature profile which has errors ranging up to about 3 per cent of the maximum temperature difference. The method is applicable to laminar boundary layers in free or forced convection; a thick boundary layer has some advantage over a thin one, in that it can "capture" more light and give a clearer ray deflection pattern due to higher intensity contrast. The method does not work well with small temperature differences, since ray deflections are then also small.

One further potential use for the optical method involves the study of oscillating boundary layers. In various studies of the response of laminar boundary layers to oscillations, solutions have been developed in which one of the predicted effects is a periodic fluctuation of temperature, and temperature gradient, superimposed on the time-average distribution within the boundary layer. Photographic observations would typically cover very many cycles, and fluctuations of temperature gradients would be expected to produce a predictable dispersion of the wire "shadow". This remark is prompted by the observation that for some of the pictures obtained when a strong sound field was present in the anechoic chamber, some wire deflection lines simply disappeared.

7. CONCLUSIONS

1. The theory of light ray deflection by a gradient of refractive index is well established, and its interpretation for the deflection of rays which pass through a thermal boundary layer over the surface of a plane or cylindrical body, with ray directions initially parallel to local isochoric surfaces, can easily be shown.
2. It is practicable to determine deflections of rays which enter a thermal boundary layer at various levels by the insertion of a suitable object in the light path optically upstream of the heated body under study.
3. Measurements of ray deflections for natural convection over a heated horizontal circular cylinder are in excellent agreement with deflections predicted from analysis.
4. Measurements of ray deflections for natural convection over a heated horizontal circular cylinder are also in excellent agreement with deflections predicted from a transcendental profile approximation function fitted to the analysis. The transcendental approximation is superior for this purpose than a polynomial of low order. The success of this approximation constitutes the major advance in the present work.
5. Straightforward techniques can be applied to determine estimations of the distribution of temperature and temperature gradient, making use of the approximation function, from ray deflection patterns.
6. The method is restricted to laminar boundary layers.

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Résumé—Des méthodes de strioscopie pour des mesures de transport de chaleur utilisent la déviation de rayons lumineux due aux gradients d'indices de réfraction. L'analyse des déviations des rayons est revue rapidement, et l'on décrit une technique expérimentale. Une comparaison entre les déviations mesurées et celles prédites théoriquement est faite pour la convection naturelle sur un cylindre horizontal chauffé, et un bon accord est obtenu. Une fonction transcendante qui approche le profil de température est décrite, et on montre que celle-ci donne aussi un bon accord quantitatif avec les déviations mesurées des rayons.

Finalement, on donne la méthode permettant d'estimer la distribution de température et le gradient de température dans une couche limite thermique à partir de mesures de déviations de rayons, en utilisant la fonction d'approximation. La technique optique est établie comme un outil pratique pour l'étude quantitative des profils de température dans les couches limites laminaires.

Zusammenfassung—Schlierenmethoden für Wärmeübergangsmessungen machen von Strahlablenkungen Gebrauch, die von Brechzahlgradienten hervorgerufen werden. Die Analyse der Strahlablenkung wird kurz rekapituliert und eine Experimentiertechnik beschrieben. Für freie Konvektion über einem waagerechten, beheizten Zylinder wird ein Vergleich zwischen gemessenen und von der Analyse vorhergesagten Strahlablenkungen durchgeführt und gute Übereinstimmung festgestellt. Es wird eine transzendente Näherungsfunktion für das Temperaturprofil beschrieben und gezeigt, dass sie ebenfalls gute quantitative Übereinstimmung mit gemessenen Strahlablenkungen ergibt. Schliesslich werden Verfahren angegeben, die das Abschätzen der Temperaturverteilung und des Temperaturgradienten in einer thermischen Grenzschicht aus Strahlablenkungsmessungen ermöglichen, wobei die Näherungsfunktion angewandt wird. Es wird festgestellt, dass die optische Messtechnik ein zweckmässiges Werkzeug für quantitative Studien von Temperaturprofilen in laminaren Grenzschichten ist.

Аннотация—При измерениях теплообмена с помощью шлирен-метода используются отклонения луча за счет изменений показателя преломления. Вкратце напоминается анализ отклонений луча и описывается методика эксперимента. Приводится сравнение измеренных и рассчитанных при анализе отклонений луча для случая естественной конвекции над горизонтальным нагретым цилиндром и между ними найдено хорошее соответствие. Описывается трансцендентная функция, приближенно описывающая температурный профиль, и показано, что она также дает хорошее количественное соответствие с измеренными отклонениями луча. Наконец, описывается методика, с помощью которой можно определить распределение температуры и температурного градиента в тепловом пограничном слое по измерениям отклонений луча, используя функцию приближения. Установлено, что оптический метод является практическим способом для количественного изучения температурного профиля в ламинарном пограничном слое.